Phys 404 Spring 2011

Homework 9, CHAPTERS 6 AND 7 Due Thursday, April 14, 2011 @ 12:30 PM

HOUR EXAM THURSDAY, APRIL 21, WILL COVER CHAPTERS 3 - 7. Once again, no notes, books, electronics, screens, phones, or calculators are allowed in the exam. You may write any formulas or calculations you wish on a single 8 ½" x 11" sheet of paper and bring it with you to the exam.

Chapter 6 assignment:

Read chapter 6, then do

- 1. K+K Chapter 6, Problem 9
- 2. K+K Chapter 6, Problem 12

Chapter 7 assignment:

Read chapter 7, then do

- 3. K+K Chapter 7, Problem 1
- 4. K+K Chapter 7, Problem 5
- **5**. Free energy of a degenerate Fermi gas I. The Sommerfeld expansion gives the energy of a low-temperature (or degenerate) Fermi gas as

$$U = U_o + \frac{\pi^2}{6} D(\varepsilon_F) \tau^2$$
 (1)

where $D(\varepsilon_F)$ is the density of states at the Fermi energy.

- a) From this result, derive an expression for the heat capacity, Eq. (37) in Chapter 7.
- b) Using the result of part a), derive an expression for the entropy.
- c) From the expression of for the energy, Eq. (1) above, and the answer to part b), show that the Helmholtz free energy of a degenerate Fermi gas can be written in the form

$$F = U_o - A \tau^2$$
 (2)

and give an explicit expression for the function A in terms of $D(\epsilon_{\scriptscriptstyle F})$.

6. Free energy of a degenerate Fermi gas II. The Sommerfeld expansion gives that the chemical potential of a degenerate Fermi gas is given by

$$\mu = \varepsilon_F - \frac{1}{3} \left(\frac{\pi}{2}\right)^2 \frac{\tau^2}{\varepsilon_F} \tag{3}$$

- a) From this expression, derive the Helmholtz free energy. (It should be the same result as part c of problem 5.)
- b) Find the pressure of a degenerate Fermi gas.
- c) Give a physical reason for the non-zero value of the pressure at $\tau=0$.